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so that the equation of the mid-diagonal line is $y = \frac{1}{2}(a + b + c + d)$. Also the centroids of the four triangles are

$$p - \frac{ab + ac + bc}{3p},\ \tfrac{2}{3}(a + b + c), \text{ etc.}$$

The equation of a parabola whose axis is praallel to the x-axis is of the form[1]

$$y^2 + Dx + Ey + F = 0,$$

and if we substitute the coördinates of any three of the centroids, evaluate D, E, F, and simplify, we obtain the equation

$$\left(y - \frac{a + b + c + d}{3}\right)^2 = -\frac{4p}{3}\left(x - p - \frac{a^2 + b^2 + c^2 + d^2}{12p} + \frac{ab + ac + ad + bc + bd + cd}{6p}\right),$$

symmetrical with respect to a, b, c, d.

The axis of this parabola is $y = \frac{1}{3}(a + b + c + d)$; therefore, the distance of the mid-diagonal line from O is three halves of the distance of this axis from O.

We may note that the four orthocenters are

$$2p,\ a + b + c + \frac{abc}{p^2}, \text{ etc.},$$

all lying on the line $x = 2p$, which is twice as far from o as the pedal line.

2791 [1919, 414; 1921, 143–145].

A cup of wine is suspended over a cup of equal capacity full of water; through a small hole in the bottom, the wine drips into the water, and the mixture drips out at the same rate. When the wine cup is empty, what part of the contents of the lower cup is water? [Proposed by Charles Gilpin, Jr., Philadelphia, as Problem 287 in *The Mathematical Visitor*, January, 1881, volume 1, page 193. No solution was published in the *Visitor*.]

REMARKS BY R. E. MORITZ, University of Washington.

Each of the solutions of the problem published in this MONTHLY (*1921*) proceeds on the assumption that the liquids flow from the cups, in other words, that the process by which the wine cup is emptied is a continuous process. This assumption seems to me unwarranted, being a flat contradiction of one of the conditions of the problem. The problem states explicitly that "the wine drips into the water, and the mixture drips out at the same rate." Dripping is essentially a discontinuous process, the solutions referred to must therefore be considered approximations rather than exact solutions.

Curiously enough, the exact solution is very much simpler than the approximations in question. Suppose that each cup contains at the outset n drops of equal size. After the first drop has fallen into the lower cup, the mixture consists of n drops of water and one drop of wine,[2] $n/(n + 1)$ of the contents of the lower cup is therefore water; after the second drop of wine has been added, the water content is $[n/(n + 1)]^2$; after the third drop has been added, $[n/(n + 1)]^3$; after the nth drop has been added, that is when the cup has been emptied, $[n/(n + 1)]^n$. It is clear then that the exact answer to the problem is a function of n, the number of drops in the wine cup at the outset.

The limit of $[n/(n + 1)]^n$ as n approaches ∞ is e^{-1}, which is the result obtained in the previous solutions. But n cannot be infinite, for if it were, the wine cup could not be emptied by a dripping process.

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[1] That the coördinates of the four centroids satisfy an equation of this type may be seen from the fact that if we substitute them all and eliminate D, E and F, we get a polynomial of degree 5 in a, b, c, d, divisible by the six differences, $a - b$, etc., and, therefore, zero—EDITORS.

[2] This assumes that the cup can hold $n + 1$ drops till they are mixed—EDITORS.